

Name of College - S.S. College J-Bad

Dept - Mathematics

Teacher's Name - Dr. Shree Nath Sharma
(Associate Prof.)

Topic - Continuity and Differentiability

Time - 12 P.M to 1.30 P.M

Date - 12-04-2021

Class - B.Sc II
(HONS)

Study Material $\rightarrow$ Concept of Continuity
From Geometrical Consideration we

derive the Concept of Continuity.

If the Graph of a function $y = f(x)$ is continuous curve, then the function $y = f(x)$ is said to be continuous function.
It means a small change in the value of x should produce a small change in the value of y and so the Graph of function $y = f(x)$ should be a continuous curve without any break in it.

But the analytical definition of Continuity is quite distinct from the geometrical concept.

Continuity of $f(x)$ at a point $\rightarrow$ A Function $f(x)$ is said to be continuous at

$x = a$ if

$$\lim_{h \rightarrow 0} f(a-h) = \lim_{h \rightarrow 0} f(a+h) = f(a)$$

Continuity of $f(x)$ in the Given ~~int~~ interval

$[a, b]$ is if

(i) $\lim_{h \rightarrow 0} f(a+h) = f(a)$

(ii) $\lim_{h \rightarrow 0} f(b-h) = f(b)$

(iii) $\lim_{h \rightarrow 0} f(c+h) = f(c)$ where $c \in (a, b)$

Discontinuity $\Rightarrow$ If $f(x)$ is not continuous at $x=a$ then it is said $f(x)$ is discontinuous at $x=a$.

Types of Discontinuities of function at a point $\rightarrow$

① Discontinuity of the First kind $\rightarrow$

A function $f(x)$ is said to have Discontinuity of the First kind at $x=a$ if $\lim_{h \rightarrow 0} f(a-h)$ and $\lim_{h \rightarrow 0} f(a+h)$ does exist but not equal.

② Discontinuity of 2nd kind $\rightarrow$

If $\lim_{h \rightarrow 0} f(a-h)$ and $\lim_{h \rightarrow 0} f(a+h)$ does not exist then $f(x)$ is said to have Discontinuity of 2nd kind.

Mixed Discontinuity $\rightarrow$ A function $f(x)$ is said to have a Mixed Discontinuity at $x=a$ if one of limit

$\lim_{h \rightarrow 0} f(a+h)$ and $\lim_{h \rightarrow 0} f(a-h)$ exists and other does not.

~~Removable~~ Removal Discontinuity $\rightarrow$
A Function $f(x)$ is said to have
a removal discontinuity at $x = a$

$$\text{if } \lim_{h \rightarrow 0} f(a+h) = \lim_{h \rightarrow 0} f(a-h) \neq f(a)$$

Jump Discontinuity $\rightarrow$

A function $f(x)$ at $x = a$ is
said to have a jump discontinuity

$$\text{if } \lim_{h \rightarrow 0} f(a+h) \neq \lim_{h \rightarrow 0} f(a-h) \text{ and}$$

$f(a)$ may be equal to neither of
the above limits.

Infinite Discontinuity $\rightarrow$ A function

$f(x)$ is said to have an infinite
discontinuity at $x = a$ if one or
more of the four functional limits

$$f(a+h), f(\underline{a+h}), f(\overline{a-h}) \text{ and } f(\underline{a-h})$$

is indefinitely large.